Square Roots

Anchors Addressed
A1.1.1.1.2 – Simplify square roots (e.g. \( \sqrt{24} = 2\sqrt{6} \)).

Lesson Pretest
Solve the following problems. Show your work in the space below and fill in the bubble that represents the correct answer.

1. An expression is shown below.
   \( \sqrt{21x} \)
   For which value of \( x \) should the expression be further simplified?
   
   A. 5
   B. 6
   C. 11
   D. 55

2. An expression is shown below.
   \( 2\sqrt{43x} \)
   Which value of \( x \) makes the expression equivalent to \( 14\sqrt{43} \)?
   
   A. 7
   B. 12
   C. 49
   D. 98

Lesson Keywords
Composite Number, Cube Root, Integer, Perfect Square, Prime Number, Radical Expression, Square Root
Square roots are the reverse operation of squaring a value. Numbers whose square roots are Integers are called **perfect squares**. For example: $\sqrt{4} = 2$, so 4 is a perfect square. The square root symbol ($\sqrt{}$) is called a radical. You can use a calculator to determine the approximate value of a square root or simplify square roots, as shown below:

### Calculating the Value of a Square Root

To estimate the value of a square root, use a calculator. Some calculators require you to enter the value first and then press the square root key (usually % $\sqrt{}$) and others require you to press the square root key and then enter the value. You will need to determine which method your calculator uses.

**Example 1**: Find the square root of $\sqrt{18}$. Round the answer to the nearest tenth.

**Solution**: Press % $\sqrt{}$ and type 18, then press **ENTER**. 

Rounding the answer to the nearest tenth, yields a final solution of 4.2.

### Test Taking Tip: Use a calculator that you know how to use. Understand how the features of the calculator work, including how to enter expressions into the calculator. Make sure your calculator has fresh batteries and is working properly prior to the day of the test.

### Simplifying Square Roots

Square roots that are not perfect squares can sometimes be simplified. To simplify a square root, factor the number inside the radical into prime factors. Each pair of the same factors can be moved to the outside of the radical. Simplify the factors on the outside of the radical by multiplying, and then do the same to the inside.

**Example 2**: Simplify $\sqrt{160}$.

**Solution**:

**STEP 1**: Factor 160 inside the radical: $\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$.

**STEP 2**: Since there are two pairs of two, one 2 from each pair can be removed from the radical (Think of it as 2 times 2 is 4 and the square root of 4 is 2, so a 2 is placed outside).

**STEP 3**: Repeat this step for all pairs, as shown: $2 \cdot 2 \sqrt{2 \cdot 5}$.

**STEP 4**: Finally, multiply the numbers outside the radical and then those inside the radical, as shown: $4\sqrt{10}$. 

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The following examples demonstrate two additional methods of determining your understanding of simplifying square roots.

**Example 3**: An expression is shown below.

\[ \sqrt{14x} \]

For which value of \( x \) should the expression be further simplified?

A. 15  
B. 23  
C. 35  
D. 37

**Solution**:

**STEP 1**: Factor the coefficient of \( x \), inside the radical: \( \sqrt{2 \cdot 7 \cdot x} \).

**STEP 2**: Factor each of the values in the answer choices.

A. \( 15 = 3 \cdot 5 \)  
B. \( 23 = 1 \cdot 23 \) (Prime)  
C. \( 35 = 5 \cdot 7 \)  
D. \( 37 = 1 \cdot 37 \) (Prime)

**STEP 3**: Compare the factors and match the factors from STEP ONE and STEP TWO. Only one of the answer choices should have one or more matching factors. This answer choice is the correct answer. In this example, both the number in the radical and choice C have a factor of 7. Therefore, C is the correct answer.

**STEP 4**: Check your work by replacing \( x \) with the value you selected and simplifying the square root, as shown below.

\[ \sqrt{14x} = \sqrt{14(35)} = \sqrt{490} = 7\sqrt{10} \]

Since \( \sqrt{490} \) can be simplified to \( 7\sqrt{10} \), the answer is correct.
Example 4: An expression is shown below.

$$2\sqrt{23x}$$

Which value of $x$ makes the expression equivalent to $10\sqrt{23}$?

A. 5
B. 10
C. 25
D. 50

Solution:

**STEP 1:** Divide the coefficient of the equivalent expression by the coefficient of the original expression.

$$10 \div 2 = 5$$

**STEP 2:** Recall that when we move a perfect square from the radical, we take the square root of the perfect square. Therefore, square the value from STEP ONE.

$$5^2 = 25$$

Therefore, the correct answer is 25, which is choice C.

**STEP 3:** Check your work by replacing $x$ with the value you selected and simplifying the square root, as shown below.

$$2\sqrt{23x} = 2\sqrt{23(25)} = 2\sqrt{575} = 10\sqrt{23}$$

Exercises

A. Estimate the values of the following square roots to the nearest tenth.

1. $\sqrt{11}$
2. $\sqrt{38}$
3. $\sqrt{92}$
4. $\sqrt{18}$
5. $\sqrt{14}$
6. $\sqrt{56}$
B. Simplify the following square roots. Use a calculator to verify your answers and make corrections as needed.

7. \( \sqrt{24} \)  
12. \( \sqrt{98} \)  
17. \( \sqrt{63} \)

8. \( \sqrt{10} \)  
13. \( \sqrt{20} \)  
18. \( \sqrt{125} \)

9. \( \sqrt{32} \)  
14. \( \sqrt{17} \)  
19. \( \sqrt{45} \)

10. \( \sqrt{18} \)  
15. \( \sqrt{112} \)  
20. \( \sqrt{12} \)

11. \( \sqrt{54} \)  
16. \( \sqrt{132} \)  
21. \( \sqrt{28} \)

C. Find a value of \( x \) that allows each expression to be further simplified. Do not use perfect squares.

22. \( \sqrt{22x} \)  
26. \( \sqrt{110x} \)

23. \( \sqrt{35x} \)  
27. \( \sqrt{60x} \)

24. \( \sqrt{42x} \)  
28. \( \sqrt{33x} \)

25. \( \sqrt{30x} \)  
29. \( \sqrt{70x} \)
D. Find the value of $x$ that makes each expression true.

30. $2\sqrt{14x} = 12\sqrt{14}$

31. $3\sqrt{51x} = 21\sqrt{51}$

32. $5\sqrt{27x} = 25\sqrt{27}$

33. $4\sqrt{33x} = 40\sqrt{33}$

34. $6\sqrt{29x} = 12\sqrt{29}$

35. $3\sqrt{22x} = 18\sqrt{22}$

36. $2\sqrt{71x} = 18\sqrt{71}$

37. $7\sqrt{120x} = 42\sqrt{120}$

E. Answer the following questions.

38. Simplify the value of $\sqrt{32}$. Explain each step.
Lesson Post Test

Solve the following problems. Show your work and fill in the bubble that represents the correct answer.

1. An expression is shown below.
\[ \sqrt{55x} \]
For which value of \( x \) should the expression be further simplified?

A. 7  
B. 12  
C. 13  
D. 15

2. An expression is shown below.
\[ 6\sqrt{63x} \]
Which value of \( x \) makes the expression equivalent to \( 18\sqrt{63} \)?

A. 3  
B. 9  
C. 12  
D. 18